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Quality of city life multiple criteria analysis

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ABSTRACT

International practice applies several urban indicators for sustainable cities (Monocle's Quality of Life Survey, Quality of Life Index (QLI), Indicators for Sustainability, European Green City Index, City Blueprint and others). These urban indicators can serve in performing integrated monitoring, assessing and recommending objectives sought by cities by different quantitative and qualitative aspects. Some of these tools can be applied to assessing a city's quality of life. One of the goals of this article is to compare several alternative methods for assessing a city's quality of life and their accuracies. A comparison was performed of the QLI and INVAR methods while conducting an analysis of comparable data from the 2012–2016 surveys on the Quality of Life in European Cities. Upon establishing the rankings of European cities by their quality of life with the assistance of the QLI and INVAR methods, an estimation of correspondence of results obtained by both methods and sensitivity analysis were performed based on a quantitative tool proposed in this paper. The obtained values of such criteria revealed a good level of congruity between the ranks obtained by employing both methods. The sensitivity analysis indicated that the results yielded by both the QLI and INVAR methods for rating the quality of life in European cities per the ever-fluctuating 2012–2016 data were similar. In other words, there was little difference between these methods for city ranking. This research also provides the INVAR method and its abilities to supplement the QLI with new functions: quantitative recommendations for cities under analysis by the indicators under analysis, optimization of indicators with consideration of indicators achieved in the quality of life area, and establishment of the values of the indicators under analysis permitting the city under analysis to raise its rating to the desired level.

1. Introduction

An entire array of organizations (IIED and WBCSD, 2002; United Nations, 2015; WCED, 1987), scholars and practitioners (Amini & Bienstock, 2014; Ben-Eli, 2012; Caradonna, 2014; Chasin, 2012; Christen & Schmidt, 2012; Elkington, 1998; Espinoza & Porter, 2011; Gerlagh, 2017; Koroneos & Rokos, 2012; Lozano, 2008; Pappas, 2012; Schilling, 2012; Zavodna, 2013) have offered concepts and definitions of sustainability. These are briefly deliberated next.

The Brundtland Report (WCED, 1987) has suggested a concept of sustainable development and a straightforward definition, which have been widely cited around the world since that time. The Brundtland Report (WCED, 1987) states that sustainable development is, "… development that meets the needs of the present without compromising the ability of future generations to meet their own needs." Later the International Institute for Environment and Development and the World Business Council for Sustainable Development (IIED and WBCSD, 2002) repeated this same concept of sustainable development and explained it even more stating, "One of the greatest challenges facing the world today is integrating economic activity with environmental integrity, social concerns and effective governance systems. The goal of that integration can be seen as 'sustainable development' and should be to maximize the contribution to the well-being of the current generation in a way that ensures an equitable distribution of its costs and benefits, without reducing the potential for future generations to meet their own needs." Such worldwide political debates have continued until now (e.g., United Nations, 2015) by specifying sustainability concepts and definitions more and more accurately.

In the opinion of Ahi and Searcy (2013), the term "sustainability" has been understood in diverse ways, fluctuating from an inter-generational philosophical point to a multi-dimensional term for business management. As stated by Glavič and Lukman (2007), various sustainability terms and their definitions are used by various scholars, practitioners and organizations, for example, green chemistry, cleaner production, pollution prevention and others. Glavič and Lukman (2007) examined fifty-one selected sustainability terms and their definitions

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and performed a semantic analysis. According to Ahi and Searcy (2013), primary sustainability concepts tended to emphasize environmental issues and later they gradually adopted a triple bottom line (i.e., environment, economic and social) method to sustainability. In compliance with Missimer, Robèrt, and Broman (2017), the huge and increasing collection of concepts, methods and tools in the sustainability area suggest a necessity for a structuring and harmonizing framework, containing a uniting and effective definition of sustainability. Ben-Eli (2012) holds the opinion that the concept of sustainability covers different major variables (population size; rate of consumption of resources; impacts on absorption capacity of sinks such as forests, oceans and soil; rates of regeneration capacities; a measure of well-being and others), all theoretically measurable.

There is no commonly approved sustainability definition. Diverse sustainability understandings can be found. As stated by Kirkby, O'Keefe, and Timberlake (1995), many authors expressed sustainable development employing at least 70 diverse definitions that were compiled by 1992. As believed by Elkington (1998), sustainability can be a 2 + 2 = 5 (or even 50) game. In accordance with Elkington (1998), to achieve outstanding triple bottom line performance, new types of economic, social and environmental partnerships are needed. In accordance with Lozano (2008), it is feasible to break down the different sustainable development definitions into the following categories: (1) conventional economists' perspective; (2) non-environmental degradation perspective; (3) integrational perspective, i.e., encompassing economic, environmental and social aspects; (4) inter-generational perspective and (5) holistic perspective. In some cases, the boundaries between perspectives may be blurred. Lozano (2008) recommends that sustainability, as an idea, is as an integrating framework – a means for seeing the relationships between various dimensions, rather than just evaluating sustainability, i.e., as a single component. Ben-Eli (2012) offers the following sustainability definition, “A dynamic equilibrium in the processes of interaction between a population and the carrying capacity of its environment such that the population develops to express its full potential without producing irreversible adverse effects on the carrying capacity of the environment upon which it depends.” Amini and Bienstock (2014) integrated various viewpoints on corporate sustainability in order to develop a multidimensional and comprehensive definition of corporate sustainability. Gerlagh (2017) defines “generous sustainability” as a combination of two conditions: neither instantaneous maximin utility nor attainable maximin utility should decrease over time.

In the opinion of King (2013), the definition of “urban development” means dissimilar things to various individuals and can be used either in one area of a town or in an entire municipal area. The definition of urban development is “the development or improvement of an urban area by building” or “an urban area that has been developed and improved by building” (Collins English Dictionary). For example, Urban Development Concept Berlin 2030 delivers an inter-agency model for the long-term, sustainable development of the city by applying a variety of strategies and goals as well as highlights the areas that will concentrate its future development. The Urban Development Concept Berlin 2030 contains a status report and strategies for Berlin 2030. The status report specifies the strengths and weaknesses as well as the opportunities and risks regarding a sustainable development of Berlin. Based on this, the strategies for Berlin 2030 emphasize the capital's developmental goals, favorable initiatives and particular districts for exemplary realization.

There are extensive efforts made to adapt the sustainability concept in the urban development context. Several terms applied for the closest connection of the sustainability with the urban development concepts are sustainable urban infrastructure, sustainable urbanism, green urban development, ecological urbanism, green urbanism, sustainable city, eco-city, zero-carbon city, sustainable cities, resilient cities and eco-municipalities. These terms can also encompass an entire array of the definitions of their composite parts, such as green building, green construction, sustainable building, natural building, ecohouse, sustainable architecture, ecological design, ecological restoration, sustainable landscape architecture, renewable energy and the like.

As stated by Ji, Li, and Jones (2017), various green urban development concepts exist in China; these are not specifically defined by standards and regulations. In the opinion of Jabareen (2006), urban sustainable forms are defined by compactness, sustainable transport, density, mixed land use, diversity, passive solar design and greening. Stossel, Kissinger, and Meir (2017) hold the opinion that the advancement of urban sustainability needs an application of different measures such as environmental policy, behavioral change and technological developments, which have to be taken at different spatial scales. Shen, Xiaoling Zhang, and Shuai (2017) analyze the efforts of sustainable urbanization by different international institutions and local governments all over the world involved in sustainable urbanization at different levels. According to Fu and Zhang (2017), sustainable city concepts, eco-cities and low-carbon cities in China represent two trends to encourage urban sustainability. In the opinion of Fu and Zhang (2017), the eco new cities are worried about the development of a sustainable way of life and a sustainable way of production with an uneven stress on economic sectors such as industrial integration and transformation.

The effort for sustainable city development is to assure a balanced development of a city and its composite parts by satisfying the well-being of its residents in the present while not harming their life styles in the future. Such a goal can be implemented by employing various social, economic and environmental methods as well as methods from other scholarly fields. The endeavors for sustainable city development are for decreasing poverty, improving the quality of life and social contacts as well as community relationships by satisfying major human needs and fostering economic and political developments that are conducive while attempting to avoid damaging the natural resources. It is possible to perceive a unity of contradictions in the sustainable development of cities, when some goals contradict others. For example, economic growth is impossible without a greater use of resources; therefore some scientists propose conserving nature by reducing consumption. Balanced economic development does not necessarily encompass the dimensions of ecological, social and cultural balanced developments. Frequently scholarly literature discusses whether a balance is possible in practice between economic, environmental and social developments of a built environment and cultural diversity. Therefore the methods of multiple criteria analyses are most suitable for analysing sustainable city developments.

In the opinions of the International Institute for Environment and Development and the World Business Council for Sustainable Development (IIED and WBCSD, 2002), what is essential in an effort to reach the goals of sustainable development involves “verifiable measures to evaluate progress and foster consistent improvement.” Hodge and Hardi (1997) claim that an obvious sustainability conceptual framework is vital for valuation objectives since it supports to detect appropriate indicators that can be adapted to a concrete context if required. Dalal-Clayton and Barry (2014) analyzed the metrics employed for the evaluation of sustainability, such as indicators, benchmarks, audits, sustainability standards and certification systems. In conformity with Shaker (2015), societies take advantage indicators as tools to deliver an exhaustive valuation of the present situation, estimate improvement and aid set for upcoming sustainable development objectives. The set of sustainability measures existing for measuring sustainable development is overwhelming to planners, researchers and politicians, thus an explanation of interrelationships, redundancy and spatial distributions is required.

Various systems and frameworks have developed globally for assessing the sustainability of a city, e.g., Monocle's Quality of Life Survey, Mercer's Quality of Living Ranking (Quality of Living Index), EIU's Global Liveability Ranking, European Green City Index, City Blueprint, European Green Capital Award, Global City Indicators Programme and Quality of Life Index. The bases for these assessment systems and frameworks for sustainable city development along with

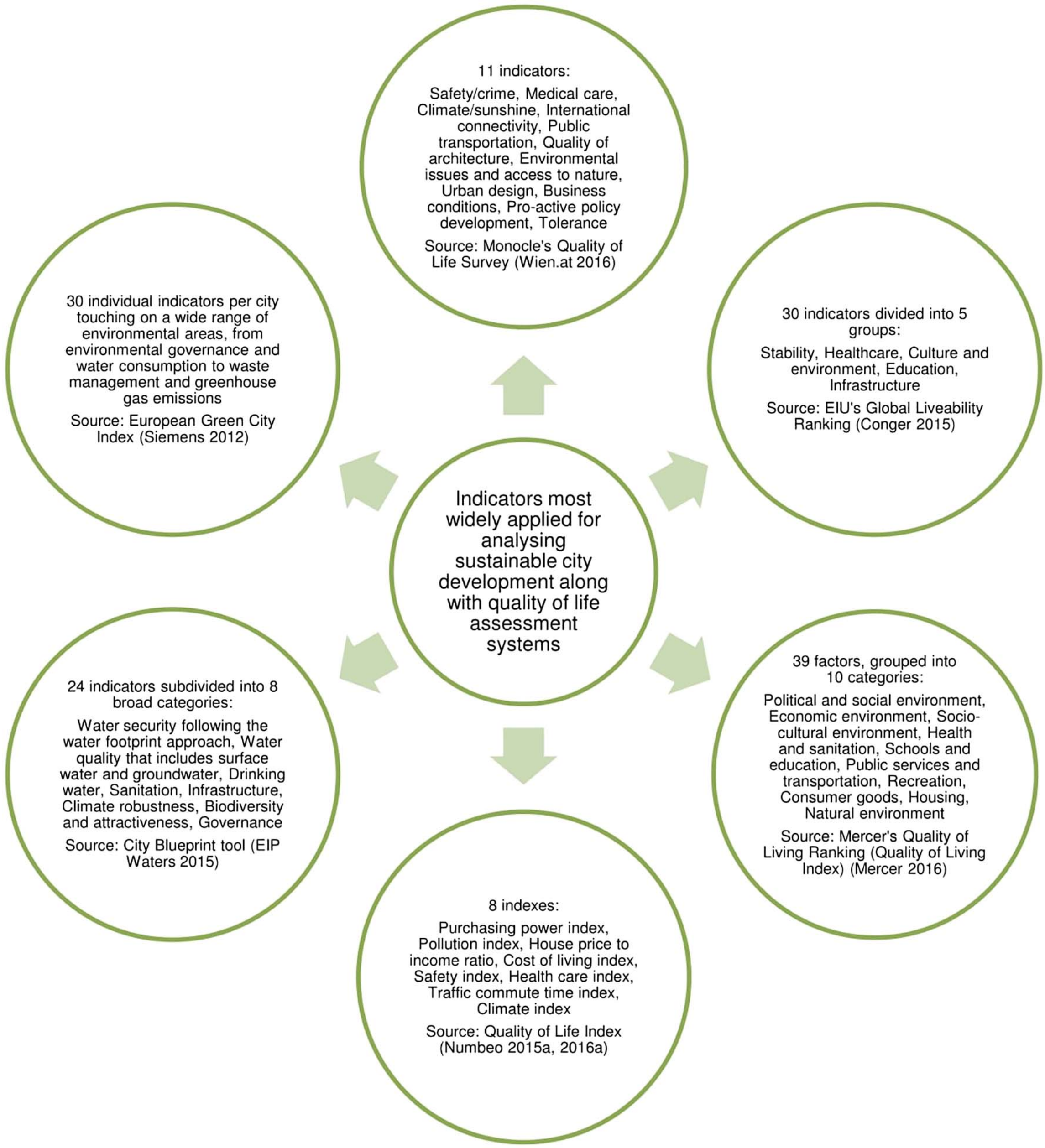

Fig. 1. Indicators most widely applied for analysing sustainable city development along with quality of life assessment systems.

the respective city's quality of life usually consist of a comprehensively defined system of indicators and quantitative and qualitative indicators describing their value and significance. An initiative in this direction is the ROCK project: Regeneration and Optimisation of Cultural Heritage in Creative and Knowledge Cities. In the framework of the ROCK project, the INVAR method of "Quality of City Life Multiple Criteria Analysis" was developed.

The indicators presented in Fig. 1 are the most widely applied for analysing sustainable city development along with quality of life assessment systems.

The values and significances of the quantitative and qualitative indicators describing a city under sustainable development and its quality of life are usually calculated for assessing the city. Such calculations are the basis for establishing priorities for the city. The European Green City Index methodology consists of three stages (Siemens, 2012): Data gathering; Indicator normalization; and Index construction. A similar three-stage methodology also applies to the Monocle's Quality of Life Survey (Wien.at, 2016), Mercer's Quality of Living Ranking (Mercer, 2016) and the EIU's Global Liveability Ranking (Conger, 2015), which are systems assessing the Quality of Life Index (QLI) (Conger, 2015). NUMBEO (Numbeo, 2015a, 2016a) developed the Quality of Life Index. The QLI is an estimation of the overall quality of life by using an empirical formula that takes into account eight different indexes (see Fig. 1) (Numbeo, 2015a, 2016a). Also Chen (2016), Lazauskaitė, Burinskienė, and Podvezko (2015), Xia, Zuo, Skitmore, Chen, and Rarasati (2015), Wei, Huang, Li, and Xie (2016), Yue, Zhang, and Liu (2016), Hsu and Juan (2016), Nuuter, Lill, and Tupenaite (2015) analyze quality of life in various aspects.

The structure of this paper is as follows. Following this introduction, Section 2 describes the INVAR method. Section 3 provides a description of the illustrative case studies. Section 4 provides a brief review of the sensitivity analysis of the congruity of the assessment results by both the NUMBEO and COPRAS (a method of multiple criteria complex proportional assessment of the projects, Zavadskas, Kaklauskas, & Sarka, 1994; Kaklauskas, 1999) methods. Finally, concluding remarks appear in Section 5.

2. INVAR method

Various multiple criteria decision-making (MCDM) methods can be applied for an analysis of the quality of city life: Analytic Hierarchy Process (AHP), Aggregated Indices Randomization Method (AIRM), Analytic Network Process (ANP), COPRAS, Disaggregation – Aggregation Approaches, ELECTRE (Outranking), Grey Relational Analysis (GRA), Goal Programming (GP), Multi-attribute Value Theory (MAVT), Multi-attribute Utility Theory (MAUT), PROMETHEE (Outranking), Technique for the Order of Prioritization by Similarity to Ideal Solution (TOPSIS), VIKOR method and others. The authors of this article have applied numerous above MCDM methods in their studies (AHP, ANP, ELECTRE, PROMETHEE, TOPSIS, VIKOR, COPRAS, etc.).

Frequently different results are obtained using different multiple criteria decision-making (MCDM) methods when solving the same problem with identical criteria and the same values and weights. Thereby a question comes up; which of these methods is the most suitable for solving some specific problem? The determination of the best multiple criteria analysis method always caused many disagreements and endless discussions. There is always an array of competing methods. It is usually very difficult to ascertain, whether the answer obtained by the application of some specific multiple criteria analysis method is correct or incorrect. Thus a sensitivity analysis was performed for this article to avoid these kinds of difficulties.

The COPRAS method has been applied sufficiently broadly in scientific research worldwide and it has been compared to other methods many times (Wang, Liu, & Quan, 2016, Qiu et al., 2015, Nuuter et al., 2015, Chatterjee, Athawale, & Chakraborty, 2011, Yin, Xiao, Wen, Qing, & Deng, 2015, etc.). These scientific studies have shown that the COPRAS method is reliable. All the data under analysis for performing the quality of life index comparisons of European cities were gathered based on the NUMBEO methodology; therefore the comparative analysis of cities was performed based on both the NUMBEO and COPRAS methods.

The INVAR method (Kaklauskas, 2016) presumes a direct and proportional dependence of the significance and priority of analyzed alternatives in a system of criteria that adequately describe the alternatives on the values and weights of those criteria. Steps 1–5 of the INVAR (see Case Study 1) are the same as in the COPRAS method (Kaklauskas, 1999; Zavadskas et al., 1994). This research also provides the INVAR method (Steps 6–10) abilities to supplement the Quality of Life Index with new functions: provision of quantitative recommendations for cities under analysis by the indicators under analysis; optimization of indicators with consideration of indicators achieved in the quality of life area and establishment of the values of the indicators under analysis permitting the city under analysis to raise its rating to the desired level.

The main steps of the INVAR method are shown in Fig. 2.

The significances and priorities of the alternatives under deliberation are calculated in the first four stages based on data from the decision matrix (alternatives, criteria values and weights). In the third stage, the significance/effectiveness (Q_j) is established for each variant under comparison, whereas, in the fourth stage – the priority of an alternative. The greater the Q_j is, the greater is the effectiveness (priority) of an alternative. The generalized criterion Q_j depends directly and proportionately on the values x_{ij} and weights q_i of the criteria under comparison.

In the fifth stage, the calculated utility degrees (U_j) of the variants under comparison directly depend on the criteria system, values and weights defining them. The effort to determine the investment value of an object under assessment that would make it equally competitive on the market involved comprehensively assessing all the positive and negative features of the objects under deliberation, which led to the recommended sixth stage for determining investment value. This stage involves calculating the investment value $x_{1j\ (\mathrm{cycle}\ e)}$ by e cycles, based on the decision matrix data (alternatives, criteria values and weights) and the utility degrees (U_j) of the alternatives, until the alternative a_j under deliberation becomes equally competitive on the market with the candidate alternatives (a_1-a_n).

The data from the decision matrix and the utility degrees (U_j) of the alternatives serve as the basis for performing the seventh and tenth stages, correspondingly as follows:

- the optimization of value x_{ij} (see Case Study 3) for any criteria during *e* approximations
- the calculation by approximation *e* cycle to determine, what the value $x_{ij\ (\mathrm{cycle}\ e)}$ should be for the alternative a_j to become the best among all the candidate alternatives (see Case Study 4)

The criteria values and weights serve as the basis for calculating the minimizing attributes S_{-j} (the lower their weight, the better is the, e.g., building price or lot price) and the maximizing attributes S_{+j} (the greater their weight, the better is, e.g., the comfortableness of the building or the aesthetics) that define the j variant. These serve as the basis for providing the quantitative recommendations in Stage 8 (see Table 5 and Case Study 2) and Stage 9 (see Case Study 2).

The next section demonstrates how the INVAR method can expand the possibilities for applying urban indicators for sustainable cities.

3. Illustrative case studies: provision of indicator frameworks assessing supplemental possibilities for city sustainability

The case studies presented next employ the Numbeo (2012, 2013, 2014, 2015a, 2015b, 2016a, 2016b), 2012–2016 mid-year quality of life data for European cities and, based on these, a comparison of the QLI and INVAR methods.

3.1. Case study 1: comparison of the quality of life in European cities, 2012–2016, by the NUMBEO and COPRAS methods

This case study presents how a city's ranking is established in terms of its quality of life and its comparison by the NUMBEO (2012, 2013, 2014, 2015a, 2015b, 2016a, 2016b) and COPRAS (Kaklauskas, 1999; Zavadskas et al., 1994) methods under same data (criteria values and weights). The performance of this analysis relied on the 2012–2016 mid-year European cities data (see Table 1). NUMBEO adjusted the calculation formulas for the Quality of Life Index as of November 2015; thereby the climate index is additionally assessed for mid-year 2016.

The establishment and comparison of life quality rankings by the NUMBEO and COPRAS methods for mid-year 2012–2016 among European cities appear numerically in Table 1 and graphically in Fig. 3. For example, the best European city assessed by the NUMBEO method was Zurich in 2013 and 2015. However, calculating by the COPRAS method, Zurich only took 4th place in 2013 and 3rd place in 2015 (Table 1, Fig. 3b, d). The calculated differences between the established rankings, while comparing the results provided by both methods, were, respectively, three rank positions in 2013 and two rank positions taking 2nd place in 2015. The calculations for the city ratings, while analysing the Berlin situation by both methods, were congruent in 2013 and in 2014 (Table 1, Fig. 3b, c). In 2013 Berlin took 2nd place in the rating and 6th place in 2014. The results of one rank position differed in 2012 and in 2015. In 2012 Berlin had taken first place according to NUMBEO and 2nd place according to the COPRAS method. In 2015 Berlin took 10th place according to NUMBEO and 11th place according to the COPRAS method. Table 1 also present sensitivity analysis results, which are described in Chapter 4.

In 52 of 66 cases, making up 69.7% of the cities, the resulting values of the sensitivity criterion appeared to be poorer than 90% but still above 80%, which indicates a rather good correspondence.

Table 2 presents the consolidation of congruities in rankings established for European cities regarding their quality of life according to the NUMBEO and COPRAS methods. For example, according to data

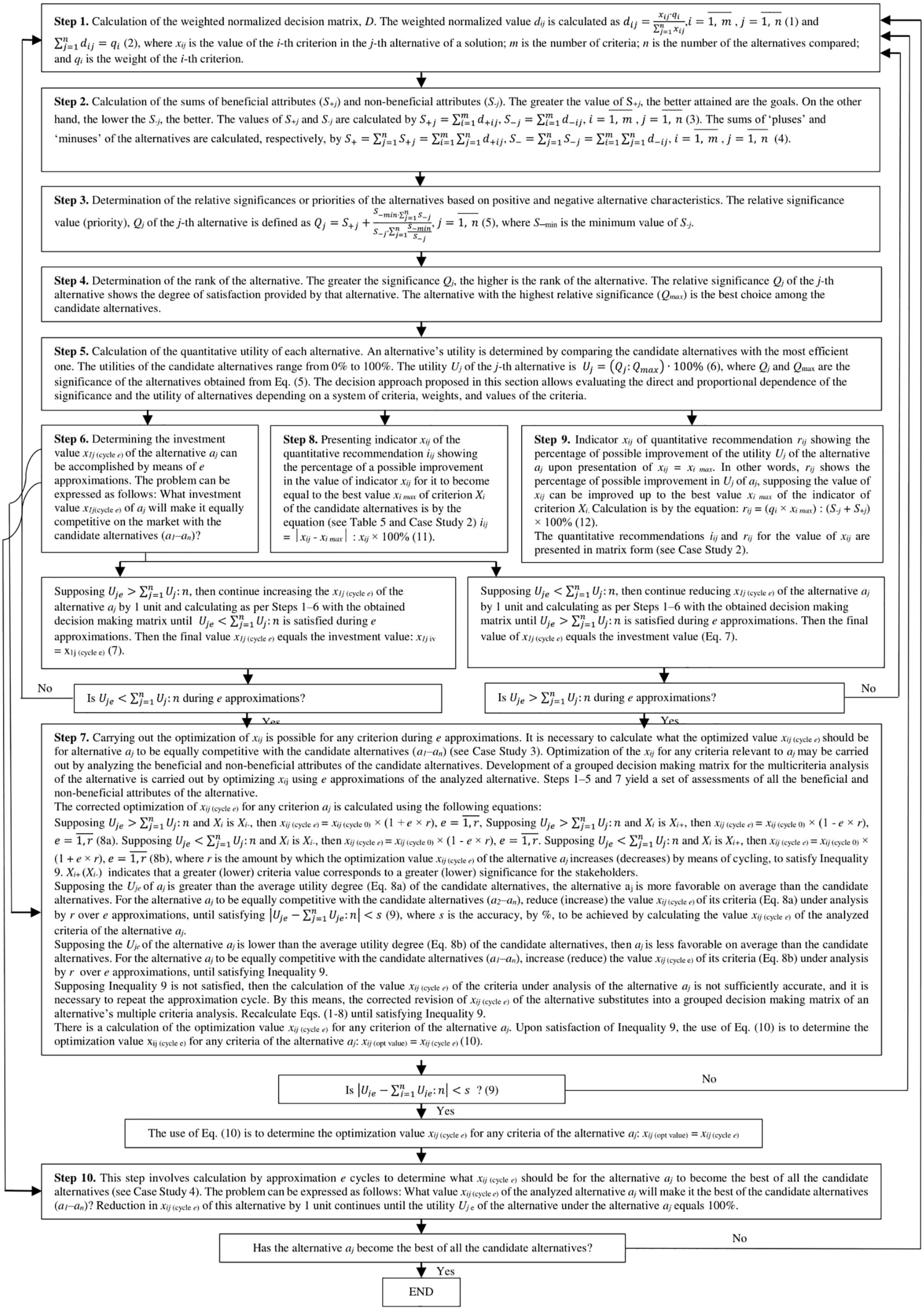

Fig. 2. The main steps of the INVAR method.

Table 1
Quality of Life Index comparisons of European cities, 2012–2016, by the Numbeo (2012, 2013, 2014, 2015a, 2015b, 2016a, 2016b) and COPRAS methods.

City No.	Year	2012			2013			2014			2015			2016			Sensitivity analysis calculation (S_j) results by %
	City	N	C	D	N	C	D	N	C	D	N	C	D	N	C	D	
1.	Zurich, Switzerland	2	3	**1**	1	4	**3**	2	3	**1**	1	3	**2**	2	4	**2**	95.44
2.	Frankfurt, Germany							1	1	**0**	2	2	**0**				100.00
3.	Munich, Germany				3	5	**2**				3	15	**12**	4	6	**2**	85.34
4.	Edinburgh, United Kingdom										4	1	**3**	1	1	**0**	95.97
5.	Trondheim, Norway	3	1	**2**	6	3	**3**	4	2	**2**	5	4	**1**				94.87
6.	Geneva, Switzerland										6	10	**4**	5	5	**0**	94.63
7.	Vienna, Austria							7	12	**5**	7	20	**13**	3	8	**5**	82.31
8.	Copenhagen, Denmark	5	4	**1**	9	7	**2**	5	5	**0**	8	7	**1**	6	5	**1**	97.23
9.	Stockholm, Sweden	4	6	**2**	8	9	**1**	8	10	**2**	9	17	**8**	13	24	**11**	85.43
10.	Berlin, Germany	1	2	**1**	2	2	**0**	6	6	**0**	10	11	**1**				98.29
11.	Trieste, Italy										11	5	**6**				89.47
12.	Glasgow, United Kingdom										12	6	**6**				89.47
13.	Helsinki, Finland										13	16	**3**	10	11	**1**	95.76
14.	Amsterdam, Netherlands										14	9	**5**	7	7	**0**	93.29
15.	Bristol, United Kingdom										15	24	**9**				84.21
16.	Hamburg, Germany				4	14	**10**				16	12	**4**	8	9	**1**	87.14
17.	Oslo, Norway				10	12	**2**	9	8	**1**	17	13	**4**	25	17	**8**	90.01
18.	Valencia, Spain										18	14	**4**				92.98
19.	Gdansk, Poland										19	18	**1**				98.25
20.	Tallinn, Estonia				11	10	**1**	10	14	**4**	20	22	**2**	17	12	**5**	92.65
21.	Ljubljana, Slovenia	7	12	**5**	15	18	**3**	15	16	**1**	21	21	**0**				92.85
22.	Sevilla, Spain										22	23	**1**				98.25
23.	Prague, Czech Republic	12	13	**1**	18	21	**3**	12	19	**7**	23	28	**5**	16	26	**10**	85.81
24.	Dublin, Ireland	6	5	**1**	13	8	**5**	13	7	**6**	24	8	**16**	24	19	**5**	80.63
25.	Brno, Czech Republic	11	11	**0**	19	15	**4**	16	15	**1**	25	29	**4**	12	13	**1**	93.90
26.	Vilnius, Lithuania				14	17	**3**	11	17	**6**	26	35	**9**	31	32	**1**	87.79
27.	Thessaloniki, Greece	20	18	**2**	32	29	**3**	23	21	**2**	27	26	**1**	20	14	**6**	92.31
28.	Porto, Portugal	9	7	**2**	17	13	**4**	14	9	**5**	28	19	**9**	14	10	**4**	87.54
29.	Cluj-napoca, Romania				26	27	**1**	19	20	**1**	29	31	**2**	22	20	**2**	96.57
30.	Lisbon, Portugal	10	9	**1**	21	16	**5**	20	18	**2**	30	30	**0**	11	18	**7**	90.70
31.	Zagreb, Croatia				28	32	**4**	21	23	**2**	31	36	**5**	23	23	**0**	92.73
32.	Madrid, Spain	8	10	**2**	16	19	**3**	24	26	**2**	32	38	**6**	15	22	**7**	89.44
33.	Brussels, Belgium	13	8	**5**	12	11	**1**	17	13	**4**	33	25	**8**	27	25	**2**	89.03
34.	Poznan, Poland										34	33	**1**				98.25
35.	Warsaw, Poland	16	19	**3**	23	23	**0**	26	25	**1**	35	40	**5**	28	31	**3**	93.06
36.	Bratislava, Slovakia				20	20	**0**				36	34	**2**	21	21	**0**	97.62
37.	Brasov, Romania										37	27	**10**				82.46
38.	Manchester, United Kingdom							18	11	**7**	38	32	**6**	18	15	**3**	88.41
39.	Wroclaw, Poland				24	26	**2**	27	22	**5**	39	37	**2**	35	35	**0**	93.78
40.	London, United Kingdom	18	20	**2**	27	31	**4**	22	27	**5**	40	46	**6**	42	46	**4**	89.70
41.	Barcelona, Spain				30	28	**2**	28	28	**0**	41	41	**0**	26	28	**2**	96.94
42.	Riga, Latvia										42	42	**0**	32	29	**3**	95.97
43.	Krakow (Cracow), Poland										43	43	**0**	33	37	**4**	94.63
44.	Paris, France	15	17	**2**	22	24	**2**	25	29	**4**	44	48	**4**	38	41	**3**	92.68
45.	Budapest, Hungary	17	14	**3**	31	22	**9**	29	24	**5**	45	39	**6**	37	33	**4**	86.49
46.	Banja Luka, Bosnia and Herzegovina				25	25	**0**	30	31	**1**	46	45	**1**				98.21
47.	Minsk, Belarus										47	50	**3**	45	45	**0**	95.97
48.	Sofia, Bulgaria	19	16	**3**	34	30	**4**	32	30	**2**	48	44	**4**	34	30	**4**	91.83
49.	Turin, Italy										49	47	**2**	30	34	**4**	94.00
50.	Milan, Italy	21	21	**0**	33	34	**1**	33	33	**0**	50	52	**2**	36	39	**3**	96.09
51.	Belgrade, Serbia	22	24	**2**	35	37	**2**	31	34	**3**	51	53	**2**	41	42	**1**	95.09
52.	Athens, Greece	25	22	**3**	35	35	**1**	34	32	**2**	52	49	**3**	39	36	**3**	94.18
53.	Bucharest, Romania	23	23	**0**	37	36	**1**	35	35	**0**	53	51	**2**	44	40	**4**	95.21
54.	Chisinau, Moldova										54	55	**1**				98.25
55.	Saint Petersburg, Russia							38	37	**1**	55	54	**1**	47	47	**0**	98.31
56.	Rome, Italy	24	25	**1**	38	39	**1**	36	38	**2**	56	57	**1**	43	44	**1**	97.04
57.	Kiev, Ukraine				39	38	**1**	37	36	**1**	57	56	**1**	49	49	**0**	98.12
58.	Moscow, Russia	26	26	**0**	40	40	**0**	39	39	**0**	58	58	**0**	48	48	**0**	100.00
59.	Reykjavik, Iceland													9	2	**7**	85.42
60.	Timisoara, Romania													19	16	**3**	93.75
61.	Novi Sad, Serbia													29	27	**2**	95.83
62.	Sarajevo, Bosnia and Herzegovina													40	38	**2**	95.83
63.	Skopje, Macedonia													46	43	**3**	93.75
64.	Bergen, Norway				5	6	**1**	3	4	**1**							97.40
65.	Belfast, United Kingdom				7	1	**6**										84.62
66.	Constanta, Romania	14	15	**1**	29	33	**4**										91.10
Congruity analysis calculation results yielded by different methods by %		91.19			91.27			91.59			90.92			91.78			

N – city ranking established by the Quality of Life Index.
C – city ranking established by the COPRAS method.
D – difference between the ranking established by the Quality of Life Index and the COPRAS method.

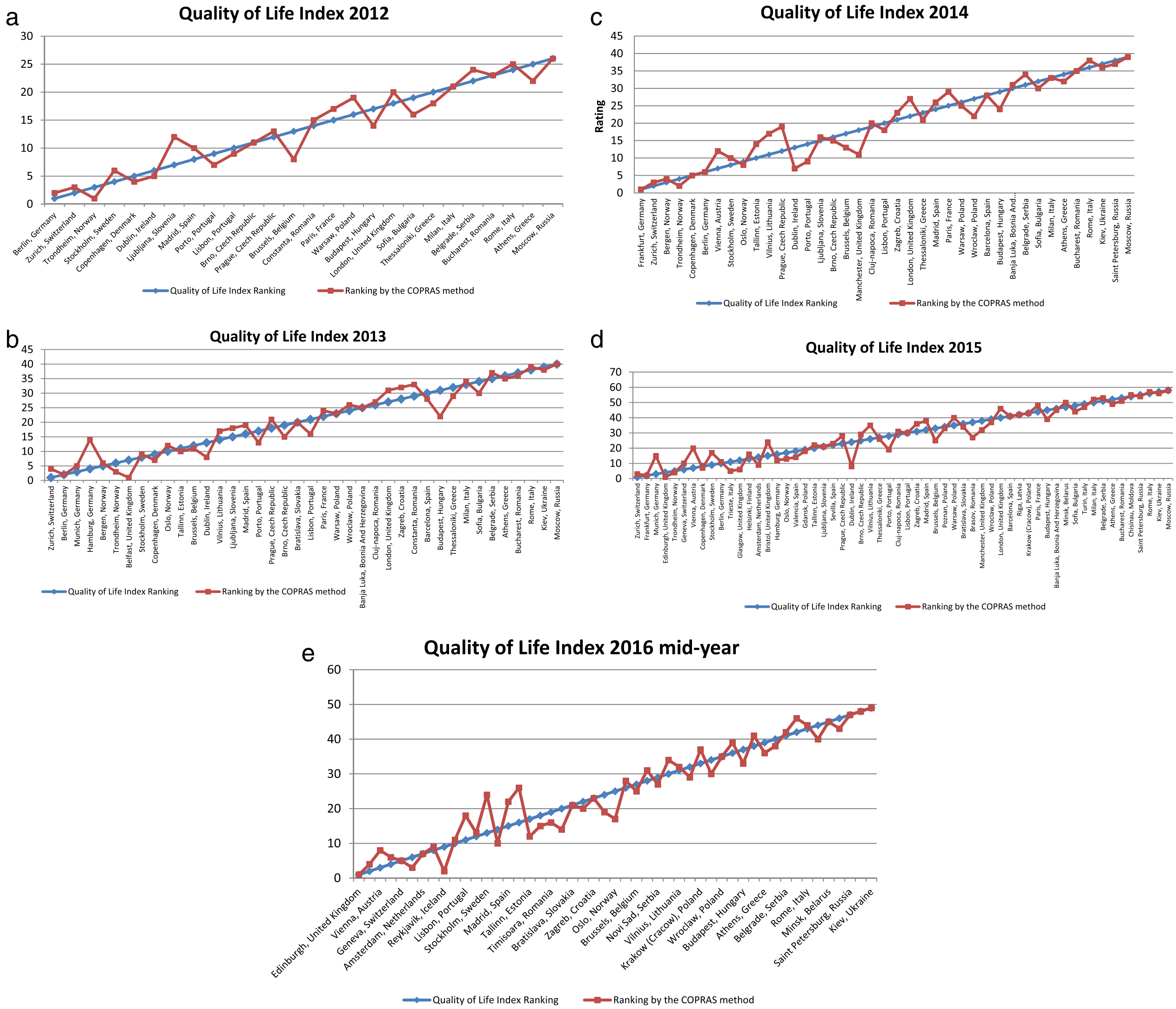

Fig. 3. Comparison of the 2012–2016 mid-year European city quality of life rankings by NUMBEO and COPRAS.

Table 2
Binned of agreements in quality of life rankings of European cities according to the NUMBEO and COPRAS methods.

Error	Incongruities by year				
	2012	2013	2014	2015	2016 midyear
Error 0	15.4%	12.50%	17.95%	12.07%	20.41%
Error by 1–5 places	84.6%	80.00%	74.36%	62.07%	65.31%
Error by 6–10 places	–	7.50%	7.69%	20.69%	12.24%
Error by > 10 places	–	–	–	5.17%	2.04%

from 2012, 15.4% of the city rankings calculated by the COPRAS method were fully congruent with the NUMBEO assessment (Brno, Milan, Bucharest, Moscow); according to 2013 data, this was 12.5% (Berlin, Bratislava, Warsaw, Banja Luka, Moscow), according to 2014 data, 17.95% (Frankfurt, Copenhagen, Berlin, Barcelona, Milan, Bucharest, Moscow) and according to 2016 mid-year data, 20.41% (Edinburgh, Geneva, Bratislava, Zagreb, Wroclaw). The city rankings in 1st–5th places, as calculated by the COPRAS and NUMBEO methods, differed for 84.6% of the cities, according to 2012 data (Berlin, Zurich, Trondheim, Stockholm, Copenhagen and others); according to 2013 data, it was for 80% of the cities (Munich, Tallinn, Brussels, Vilnius, Prague and others) and, according to 2014 data, for 74.36% of the cities (Trondheim, Stockholm, Vienna, Oslo, Zagreb and others).

3.2. Case study 2: providing quantitative tips for improving specific quality of life indicators by the INVAR method (Kaklauskas, 2016)

Eqs. (11) (Step 8) and (12) (Step 9) can serve as bases for calculating and providing quantitative tips to improve specific indicators of the quality of life. The provision of recommendations is in matrix form (see Table 3). One example is the analysis of the 2015 Pollution Index in Vienna. World practice indicates that there is a strong connection between the quality of life and pollution. For example, (Darçın, 2014) examined the relation between air quality and quality of life by using canonical correlation analysis. The data for that study was collected from 27 countries. It found a significant positive correlation between air quality and quality of life (Darçın, 2014). The aim of Sommar et al. (2014) was to evaluate the impact of traffic pollution (studied as NO_2 and NO_x) on the quality of life of asthmatic subjects and individuals with chronic rhinosinusitis (CRS) and pollution controls. Traffic pollutants have been found to be associated with the development of asthma 3–6 and CRS. Asthma and CRS substantially affect the quality of life (Sommar et al., 2014).

The data presented in Table 3 show that the least polluted city in 2015 was Stockholm (a_9) Sweden (Pollution Index x_{79} = 12.38). For

Table 3
Quantitative recommendations (a fragment) provided in matrix form for improving specific quality of life (Numbeo, 2015b).

Criteria describing the alternatives	*	Measuring units	Weight	Compared alternatives / Possible improvement of the analysed criterion by % / Galimas alternatyvos rinkos vertės padidėjimas %, įtakojamas pirmiau padidėjusios kriterijaus vertės												
				Zurich, Switzerland	Frankfurt, Germany	Munich, Germany	Edinburgh, United Kingdom	Trondheim, Norway	Geneva, Switzerland	Vienna, Austria	Copenhagen, Denmark	Stockholm, Sweden	Berlin, Germany	Trieste, Italy	Glasgow, United Kingdom	Helsinki, Finland
Purchasing Power Index	+	Points	1	133,91 (4,21%) (0,7079%)	139,55 (0%) (0%)	99,64 (40,05%) (6,7318%)	96,34 (44,85%) (7,5381%)	93,45 (49,33%) (8,291%)	130,21 (7,17%) (1,2056%)	100,22 (39,24%) (6,5956%)	96,68 (44,34%) (7,4525%)	105,62 (32,12%) (5,3991%)	97,08 (43,75%) (7,3525%)	86,19 (61,91%) (10,405%)	99,32 (40,51%) (6,8076%)	91,36 (52,75%) (8,8651%)
Safety Index	+	Points	0,75	80,34 (6,25%) (0,7876%)	70,04 (21,87%) (2,7571%)	85,36 (0%) (0%)	71,96 (18,62%) (2,3472%)	81,69 (4,49%) (0,5663%)	62,15 (37,35%) (4,7074%)	69,48 (22,86%) (2,8809%)	73,2 (16,61%) (2,094%)	54,02 (58,02%) (7,3129%)	67,45 (26,55%) (3,347%)	82,26 (3,77%) (0,475%)	47,34 (80,31%) (10,1234%)	64,37 (32,61%) (4,1103%)
Health Care Index	+	Points	0,5	71,35 (28,76%) (2,4168%)	63,1 (45,59%) (3,8315%)	90,59 (1,41%) (0,1187%)	68,19 (34,73%) (2,9182%)	86,24 (6,53%) (0,5486%)	60 (53,12%) (4,4636%)	80,79 (13,71%) (1,1525%)	74,49 (23,33%) (1,9607%)	69,44 (32,3%) (2,7144%)	75 (22,49%) (1,8902%)	73,46 (25,06%) (2,106%)	91,87 (0%) (0%)	71,37 (28,72%) (2,4137%)
Cost of Living Index	-	Points	0,2	141,06 (75,27%) (2,5299%)	83,53 (58,23%) (1,9573%)	84,16 (58,54%) (1,9678%)	91,74 (61,97%) (2,083%)	119,93 (70,91%) (2,3835%)	145,18 (75,97%) (2,5535%)	77,02 (54,7%) (1,8387%)	105,72 (67%) (2,252%)	89,15 (60,86%) (2,0458%)	74,72 (53,31%) (1,7918%)	85,24 (59,07%) (1,9855%)	92,77 (62,39%) (2,0972%)	94,2 (62,96%) (2,1164%)
Property Price to Income Ratio	-	Points	2	8,07 (39,9%) (13,4121%)	5,76 (15,8%) (5,3105%)	13,09 (62,95%) (21,1593%)	5,62 (13,7%) (4,6054%)	7,65 (36,6%) (12,303%)	10,13 (52,12%) (17,5201%)	13,3 (63,53%) (21,3559%)	6,78 (28,47%) (9,5684%)	11,88 (59,18%) (19,8908%)	7,49 (35,25%) (11,8477%)	5,47 (11,33%) (3,8099%)	5,18 (6,37%) (2,1414%)	10,94 (55,67%) (18,7117%)
Traffic Commute Time Index	-	Points	0,5	26,44 (35,7%) (3,0003%)	27,17 (37,43%) (3,1455%)	24,5 (30,61%) (2,5725%)	21,43 (20,67%) (1,7371%)	19,14 (11,18%) (0,9396%)	17,6 (3,41%) (0,2865%)	24,86 (31,62%) (2,6569%)	30 (43,33%) (3,6415%)	35,56 (52,19%) (4,386%)	42 (59,52%) (5,002%)	20 (15%) (1,2605%)	47,67 (64,34%) (5,4066%)	27 (37,04%) (3,1124%)
Pollution Index	-	Points	1	18,79 (34,11%) (5,7334%)	35,85 (65,47%) (11,0029%)	21,82 (43,26%) (7,2711%)	14,01 (11,63%) (1,9554%)	21,8 (43,21%) (7,2624%)	23,81 (48,01%) (8,0681%)	25,02 (50,52%) (8,4907%)	28,82 (57,04%) (9,5872%)	12,38 (0%) (0%)	28,61 (56,73%) (9,5342%)	42,15 (70,63%) (11,8704%)	26,25 (52,84%) (8,8804%)	19,83 (37,57%) (6,3142%)

*- The sign + (-) indicates that a greater (lesser) criterion value corresponds to a greater (lesser) significance for stakeholders

example, an effort to lessen the level of pollution in Vienna (a_7) (Pollution Index x_{77} = 25.02) down to the level in Stockholm would require various means to lower the Pollution Index in Vienna by 50.52% (i_{77} = 50.52%, Eq. (11), Step 8 serves as the basis for this calculation) (see Table 3). In this case, the quality of life in Vienna would increase by 8.4907% (r_{77} = 8.4907%, Eq. (12), Step 9 serves as the basis for this calculated amount) (see Table 3). Analogous analyses regarding the Pollution Index in other cities and its impact on their quality of life can be undertaken.

3.3. Case study 3: optimization of city criteria values

The quality of life in a city depends on its property prices according to Numbeo (2012, 2013, 2014, 2015a, 2015b, 2016a, 2016b). On their own, property prices depend on the price to income ratio, the gross rental yield in the city's center, the gross rental yield outside the city's center, the price to rent ratio in the city's center, the price to rent ratio outside the city's center, mortgage as a percentage of income, and an affordability index. In what follows, the analysis will be of the influence of price-to-income ratios, as an example, on the city's quality of life.

To achieve a quality of life for a city under analysis that is equal to that of a city under comparison, use Steps 1–5 and 7 of the INVAR method as the basis for optimizing the value of any selected criterion of the city under analysis. An analysis of the 2013 Stockholm Property Price to Income Ratio score optimization can serve as an example. The purpose of this case study is to establish what the Stockholm (a_8) Property Price to Income Ratio score ($x_{58\ \text{cycle}\ e}$) should be for the utility degree of this city's quality of life to be equal to the quality of life utility degree for Copenhagen (a_9), with consideration of minimizing and maximizing criteria.

The Stockholm (a_8) Property Price to Income Ratio score is 11.12 in the matrix of primary data from the 2015 Quality of Life in European Cities analysis. Meanwhile its utility degree equals 61.85% (Table 4). The source for the data for this calculation is from NUMBEO (Numbeo, 2015b).

The goal here is to have an approach to optimizing (in this case, lessening) the hypothetical Property Price to Income Ratio score $x_{58\ \text{cycle}\ e}$ for Stockholm (a_8) in order for the utility degree of Stockholm (a_8) to be equal to that for Copenhagen (a_9), i.e., for the quality of life to be the same in both cities. Table 4 shows that once the value of Property Price to Income Ratio lessens to 8.6, the utility degree for Stockholm's quality of life becomes nearly equal to the utility degree for Copenhagen's (it only differs by 0.04%). Table 4 also shows that after 24 approximation cycles, Inequality 5 was not satisfied ($x_{58\ \text{cycle}\ 24}$ = 8.7 $|-0.27\%|$ > 0.1%). However, after recalculating for 25 approximation cycles, the hypothetical Property Price to Income Ratio

Table 4
Calculations for the hypothetical Property Price to Income Ratio score ($x_{58\ \text{cycle}\ e}$) for Stockholm (a_8) to approach equality with the utility degree of Copenhagen (a_9).

Approximation cycle	Score $x_{58\ \text{cycle}}$	Utility degree of the Quality of Life		*
		Stockholm, Sweden	Copenhagen, Denmark	
0	11.12	61.85%	67.62%	$\lvert -5.77 \rvert$ > 0.1%
–	–	–	–	
11	10	64.19%	67.56%	$\lvert -3.37\% \rvert$ > 0.1%
–	–	–	–	
21	9	66.63%	67.71%	$\lvert -1.08\% \rvert$ > 0.1%
–	–	–	–	
24	8.7	67.44%	67.71%	$\lvert -0.27\% \rvert$ > 0.1%
25	8.6	67.69%	67.65%	$\lvert 0.04\% \rvert$ < 0.1%

* Inequality 5 is used to determine whether the calculation of the revised value $x_{58\ \text{cycle}\ e}$ of under valuation a_8 is sufficiently accurate.

score for Stockholm (a_8) was lessened to 8.6, and the utility degree of the quality of life for this city equalled that of Copenhagen ($x_{58\ \text{cycle}\ 25}$ = 8.6 $|0.04\%| < 0.1\%$), i.e., the quality of life became the same in both cities.

3.4. Case study 4: what should the hypothetical property price to income ratio score be for Vilnius for it to land among the top 10 European cities in the Quality of Life Index?

In the opinion of Numbeo (2012, 2013, 2014, 2015a, 2015b, 2016a, 2016b), the level of the quality of life for cities correlates with a city's affordability and its price to income ratio. For example, increased demand for housing leads to higher prices and lower affordability. Strong demand bids housing costs up in nice places to live. The opposite is true as well. Regions with underperforming economies and a lower quality of life do have better affordability. A strong regional economy and high quality of life do come at the cost of lower housing affordability (Lehner, 2016). The price to income ratio is a straightforward way to evaluate the affordability of housing in a specific zone. Rosen (1979) and Roback (1982) extend this model to consider the relation between wages and rents to measure market-based differences in the quality of life across cities.

The goal of this case study is to establish the hypothetical Property Price to Income Ratio score ($x_{514\ \text{cycle e}}$) for Vilnius (a_{14}) so that Vilnius would land among the top ten assessed European cities (a_1–a_{40}) in the Quality of Life Index, with consideration of minimizing and maximizing criteria. Here is an analysis of the 2013 data, as an example Numbeo (2015b). According to the 2013 data, the Property Price to Income Ratio score for Vilnius was 13.74. The calculations were performed establishing that the utility degree of the quality of life in Vilnius is 52.61% (17th place). For the rating of Vilnius to rise by at least seven places and land among the top ten European cities in the Quality of Life Index, the value of the Property Price to Income Ratio ($x_{514\ \text{cycle e}}$) must be sufficiently lessened. Applications of Steps 1–5 and 10 of the INVAR method were used to perform the calculations. Table 5 presents these calculations.

Table 5 shows that after 37 approximation cycles, Vilnius was in the 13th place on the Quality of Life Index in European Cities. Therefore, since the desired result had not been reached yet, the hypothetical Property Price to Income Ratio Score was further lessened. After 47 approximation cycles, the Property Price to Income Ratio score for Vilnius fell by a factor of 1.528, and the utility degree of Vilnius was at 63.47%. Now it did land among the top ten European cities in the QLI.

There is a comparison of the Quality of Life Index and the INVAR method to establish the accuracy of the suggested INVAR method. The assessment of a city's quality of life employs the same 2012–2016 data on European cities (criteria values and significances). Proprietary quantitative evaluation criteria were proposed and employed for this paper in order to gauge the congruity between the ranking results obtained by using the COPRAS and NUMBEO evaluation methods. The values obtained for such criteria appeared to be around 90%. Such results revealed a good level of congruity between the ranks obtained by both methods. The performance of a sensitivity analysis of the assessment yielded similar results.

Table 5
What should the value of the property price to income ratio be for Vilnius to land among the top 10 European cities in the Quality of Life Index?

Approximation cycle	Property Price to Income Ratio Score, $x_{514\ \text{cycle e}}$	Utility degree of Vilnius's (a_{14}) quality of life	Rating
0	13.74	52.61%	17
–	–	–	–
7	13	53.91%	15
–	–	–	–
37	10	60.56%	13
–	–	–	–
47	9	63.47%	9
–	–	–	–
67	7	70.56%	5

The systems and frameworks for assessing sustainable city developments and their quality of life can be enhanced with certain supplementary possibilities, e.g., those that the INVAR method provides. An analysis appears later in this article regarding such new possibilities.

The aforementioned frameworks of indicators assessing the sustainability of cities do not provide automated quantitative guidelines for enriching the concrete indicators. They are unable to rationalize designated indicators by considering the existing quality of life situation in the city under analysis; they are unable to calculate the values of indicators which would permit a city to be best of the others under analysis. INVAR method can be used for above purposes.

4. Sensitivity analysis of the congruity of results from the assessments by both the NUMBEO and the COPRAS methods

Proprietary quantitative assessment criteria were proposed and used for this paper in order to evaluate the congruity between ranking results obtained by using the COPRAS and the NUMBEO evaluation methods. Such criteria can be used for gauging levels of congruity between ranks obtained by any set of MCDA methods. The use of such criteria here revealed a solid level of congruity between the COPRAS and NUMBEO methods in terms of their ranking results.

Apply the equation of standard error estimation (13) to compare results yielded by the NUMBEO and COPRAS methods:

$$S_Y = \sqrt{\frac{\Sigma_{j=1}^{n_Y}(N_{jY} - Q_{jY})^2}{n_Y}}, \tag{13}$$

where N_{jY} is the rank of the j-th city yielded by the NUMBEO method and Q_{jY} is the rank of the j-th city yielded by the COPRAS method for the year Y; n_Y is the total number of assessed cities for which data is available during year Y. Min S_Y is zero. Such a value is attained when the results obtained by both methods are congruent. Max S_Y is attained when the difference between the results obtained by both methods is maximal, i.e., when the difference equals $n_Y - 1$. As, for every j,

$$N_{jY} - Q_{jY} = n_Y - 1, \tag{14}$$

$$max\ S_Y = \sqrt{\frac{\sum_{j=1}^{n_Y}(n_Y - 1)^2}{n_Y}} = n_Y - 1. \tag{15}$$

The next index gauges the congruity between the results obtained by the two methods for the year Y, which is obtained as follows:

$$R_Y = \left(1 - \frac{S_Y}{\max S_Y}\right)100\%. \tag{16}$$

In case the standard error is the largest possible and does not differ from maxS_Y, the index R_Y will show 0% correspondence, whereas if no difference between ranks was obtained using the two methods, the index would be 100%. In other words, the smaller is the standard error, which comprises the magnitude of all differences between ranks obtained in the solution, the closer is the proposed index to 100%.

Table 6 presents the values of S_Y, max S_Y, and R_Y for each year of the observed period.

Table 6
Criterion R_Y values congruent with the results, %.

	2012	2013	2014	2015	2016
S_Y	2.20	3.41	3.19	5.17	3.94
max S_Y	25	39	38	57	48
R_Y	91.19%	91.27%	91.59%	90.92%	91.78%

The values resulting from criterion R_Y (bottom row of Table 6) show convincing levels of congruity between the results obtained by the two methods in question.

For a sensitivity analysis, variation of data should be induced or data with variation taken. The latter approach is undertaken in this paper by taking varying values of criteria in the 5-year investigated period for each city separately. The ranking of the cities also varied over the period. A similar criterion for gauging sensitivity is taken for each city to observe the comparability between the methods for varying data. The denotation of A_j references the total number of years for which data is available for city *j*. The equation for the criterion to gauge congruity between ranks obtained by two methods over A_j years is the following Eq. (17):

$$R_j = \left(1 - \frac{S_j}{\max S_j}\right)100\%, \tag{17}$$

where

$$S_j = \sqrt{\frac{\Sigma_{Y=1}^{n_Y}(N_{jY} - Q_{jY})^2}{n_j}}. \tag{18}$$

Again, N_{jY} is the rank obtained by the NUMBEO method of the *j*-th city for the *Y*-th year, while Q_{jY} is the rank obtained by the COPRAS method of the *j*-th city for the *Y*-th year. Min $S_j = 0$ is attained, whenever ranks obtained by the two methods appear to be the same:

$$N_{jY} = Q_{jY}\ (j = 1,\ldots, n_j\ ). \tag{19}$$

Max $S_j = n_Y - 1$ is attained, when differences between ranks obtained by the two methods are the largest for every year *Y*:

$$N_{jY} - Q_{jY} = n_Y - 1 \quad \forall\ Y. \tag{20}$$

Consequently

$$\max\ S_j = \sqrt{\frac{\sum_{Y\in ¥_j}^{n_j}(n_Y - 1)^2}{n_j}}, \tag{21}$$

where $¥_j$ is the set of those years when data were available for the *j*-th city and included in the research. The set of values of max S_j corresponds to the set of available combinations of "non-zero" years; its values depend on the corresponding combination.

The values of the sensitivity criterion S_j are presented in Table 1 (the right-hand side column). The calculated values of criterion S_j appear to be around 90%, which reflects a rather strong stability of results for the data for all 66 cities. The diagram in Fig. 4 depicts those results whose criterion values are smaller than 90%. These are shown in yellow. The average of S_j measures the resulting average congruity. It appears to be 94.11%, indicating a rather strong stability over the period under investigation.

For cities with five years of available data, there is a better reflection of the sensitivity of the criterion, since it contains more information on the differences between rankings.

In 47 cases of 66, or 71.1% of the total number of cities, the resulting values of the sensitivity criterion appeared to be higher than 90%. In all other cases, the values were lower than 90% but still above 80%, indicating a rather strong congruity.

5. Conclusions and future work

Various frameworks for assessing indicators of city sustainability are employed worldwide, including Urban Sustainability Indicators, Mercer's Quality of Living Ranking, Monocle's Quality of Life Survey, European Green City Index, Quality of Life Index, Global City Indicators Programme, City Blueprint, Indicators for Sustainability, and the EIU's Global Liveability Ranking. These frameworks determine quality of life values in their assessments. Nonetheless, for now, these frameworks for assessing indicators of city sustainability do not provide automated quantitative guidelines for enriching concrete indicators. They are unable to rationalize designated indicators by considering the existing quality of life situation in the city under analysis; they are unable to calculate the values of the indicators which would permit a city to be best of the others under analysis. The INVAR method supplements the frameworks with new above functions.

There is a comparison of the Quality of Life Index and the INVAR method to establish the accuracy of the suggested INVAR method. The assessment of a city's quality of life employs the same 2012–2016 data on European cities (criteria values and weights). Usually all the frameworks for gauging indicators of a city's sustainability have developed ranking grades that are diverse, to some extent.

Table 6 presents the values of the quantitative sensitivity criteria proposed in this article. It reveals how a variation in criteria values affects the magnitude of the discrepancy between resulting city rankings upon application of these two different techniques. Researches have shown that, as data fluctuate, sensitivity in the differences for establishing a city's prioritization according to both methods under analysis was low. The examination of the variation in indicator values did not have any effect whatsoever on the ranking of cities upon applying Quality of Life Index and INVAR techniques. An estimation of correspondence of results obtained by both methods and sensitivity analysis were performed based on the proposed in this paper quantitative tool. The sensitivity analysis revealed that the QLI and INVAR

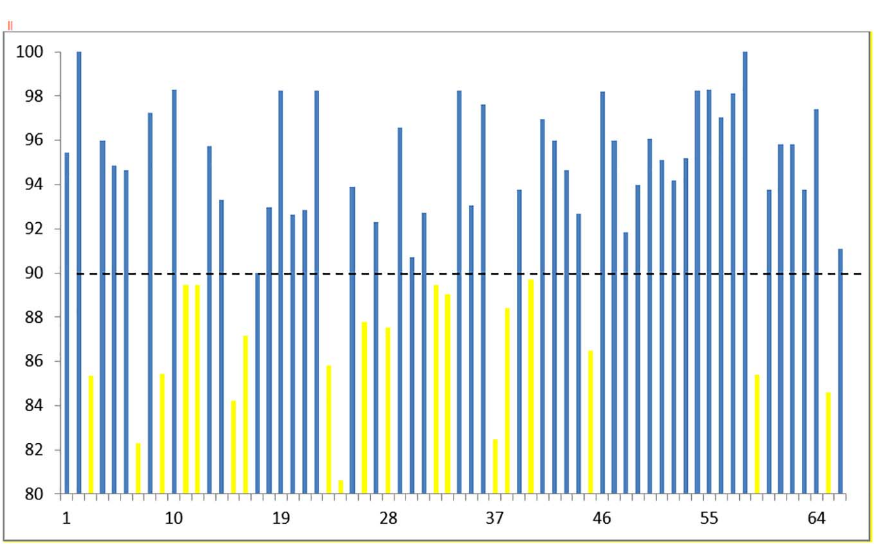

Fig. 4. Criteria values to gauge congruity between results obtained by the COPRAS and NUMBEO methods for ranking 66 cities.
Legend:
Blue – shows those results whose sensitivity criterion S_j values are higher than 90%.
Yellow – depicts those results whose sensitivity criterion S_j values are smaller than 90%.
- - - - - – the average of S_j measures the resulting average congruity. It appears to be 94.11%. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

methods tolerated changes in the 2012–2016 indicator values, which have an insignificant ranking effect on cities after applying these two techniques. Standard error estimates demonstrate undoubtable levels of conformity between the rankings obtained by the two techniques under analysis. Studies have also shown that the INVAR method is totally free of the rank reversal phenomenon, which is recognized as a key basis for encountering accuracy difficulties with assessments. Additionally, the INVAR method is presented in this article along with its potential for supplementing frameworks of indicators assessing city sustainability, e.g., the QLI. The recommended INVAR method will be useful in the practice of scientists in various fields and governmental and non-governmental institutions. Future work with the INVAR method will involve a comparison of the aforementioned urban indicators for sustainable cities leading to recommendations on practical opportunities for expanding the potential of these tools.

City planners can improve the quality of life of a town in the four directions submitted next if they decide to apply the INVAR Method.

Often city planners determine possibilities for the economic development and growth of a city and support investments in a district. Therefore a city planner could determine the investment value of a project under deliberation with the INVAR Method.

Furthermore the INVAR Method can contribute in giving numerical tips for improving operations in the area of city planning (land use, strategic city, regional, master, transportation, environmental and infrastructure planning, city renewal and design, heritage and conservation).

The INVAR Method can assist in rationalizing (calculating the specific size of a field's weight) a designated field of city planning (land use, strategic city, regional, master, transportation, environmental and infrastructure planning, city renewal and design, heritage and conservation). This would involve looking for how the field under analysis would be similarly rational in the city as compared to the other cities under comparison. The same rationalization can also be performed for the composite parts of that field of city planning. For example, a city planner performing the design of a city would be able to optimize public spaces (parks, squares, streets), the infrastructure (hospitals, schools, electricity, transport infrastructure, heating, water supply) or environmental planning (flora, fauna, water) by comparisons with other analogical parts of the city.

The INVAR Method can assist in calculating the size of the weight of some specific field of city planning, so a town under analysis can become the best among towns under deliberation. For example, a city planner could analyze the alternatives and their composite parts of the heritage and conservation of an old town supported by UNESCO, so the old town under analysis can become the best among similar other towns under analysis.

The four directions submitted above demonstrate the innovative insights of the research, which could generate the conditions for potential managerial implications of the study to modify the approach for the thought and behavior of a city planner.

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